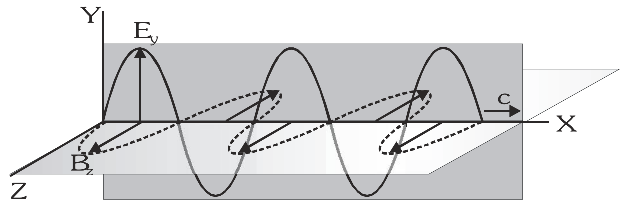
ELECTROMAGNETIC WAVES

INTRODUCTION

A changing electric field produces a changing magnetic field and vice versa which gives rise to a transverse wave known as electromagnetic waves. The time varying electric field and magnetic field mutually perpendicualr to each other are also perpendicualr to the direction of propogation.

Thus the electromagnetic waves consist of sinusoidally time varying electric and magnetic field acting at right angles to each other as well as at right angles to the direction of propogation.



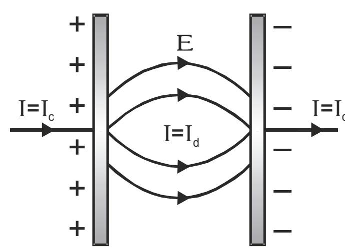
HISTORY OF ELECTROMAGNETIC WAVES

- In the year 1865, Maxwell predicted the electromagnetic waves theoretically. According to him, an accelerated charge sets up a magnetic field in its neighborhood.
- In 1887, Hertz produced and detected electromagnetic waves experimentally at wavelength of about 6m.
- Seven year later, J.C. Bose became successful in producing electromagnetic waves of wavelength in the range 5mm to 25mm.
- In 1896, Marconi discovered that if one of the spark gap terminals is connected to an antenna and the other terminal is earthed, the electromagnetic waves radiated could go upto several kilometers.
- The antenna and the earth wires are connected to the two plates of a capacitor which radiates radio frequency waves. These waves could be received at a large distance by making use of an antenna earth system as detector.
- Using these arrangements; in 1899 Marconi first established wireless communication across the English channel i.e., across a distance of about 50 km.

CONCEPT OF DISPLACEMENT CURRENT

When a capacitor is allowed to charge in an electric circuit, the current flows through connecting wires. As capacitor charges, charge accumulates on the two plates of capacitor and as a result, a changing electric field is produced across between the two plate of the capacitor.

According to maxwell changing electric field intensity is equivalent to a current through capacitor known as displacement curent (I_d). If + q and − q be the charge on the left and right plates of the capacitor respectively at any instant if σ be the surface charge density of plate of capacitor the electric field between the plate is given by



$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A}$$

If charge on the plates of the capacitor increases by dq in time dt then dq = I dt

change in electric field is $dE = \frac{dq}{\epsilon_0 A} = \frac{Idt}{\epsilon_0 A} \Rightarrow \frac{dE}{dt} = \frac{I}{\epsilon_0 A}$

$$I = \epsilon_0 A\frac{dE}{dt} = \epsilon_0 \frac{d}{dt}(EA) = \epsilon_0 \frac{d\phi_E}{dt} \quad (\because \phi_E = EA) \qquad I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

The conduction current is the current due to the flow of charges in a conductor and is denoted as Ic and **displacement current** is the current due to changing electric field between the plate of the capacitor and denoted as I_d so the total current I is sum of I_c and I_d i.e. $I = I_c + I_d$

Ampere's circuital law can be written as

$$\oint \vec{B}.d\vec{\ell} = \mu_0 (I_c + I_d) \qquad \Rightarrow \qquad \oint \vec{B}.d\vec{\ell} = \mu_0 \left(I_c + \epsilon_0 \frac{d\phi_E}{dt}\right)$$

MAXWELL'S EQUATION

There are four maxwel' equation given below

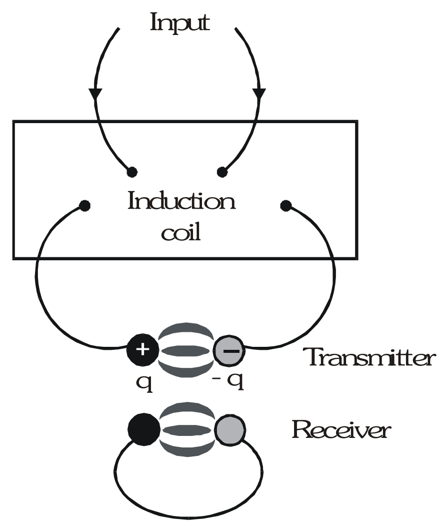
(1) Gauss law in electrostatics : $\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$ ...(i)

(2) Gauss law in magnetism : $\oint \vec{B}.\vec{ds} = 0$ ...(ii)

(3) Faraday's law of electromagnetic induction : emf = $\oint \vec{E}.\vec{d\ell} = -\frac{d\phi_B}{dt}$ ...(iii)

(4) Maxwell - Ampere's circuital law : $\oint \vec{B}.\vec{d\ell} = \mu_0 \left[I_c + \epsilon_0 \frac{d\phi_E}{dt} \right]$ ... (iv)

HERTZ EXPERIMENT (Practical production of EM waves)



- In 1888, Hertz demonstrated the production of electromagnetic waves by oscillating charge. His experimental apparatus is shown schematically in fig.
- An induction coil is connected to two spherical electrodes with a narrow gap between them. It acts as a transmitter. The coil provides short voltage surges to the spheres making one positive and the other negative. A spark is generated between the spheres when the voltage between them reaches the breakdown voltage for air. As the air in the gap is ionised, it conducts more rapidly and the discharge between the spheres becomes oscillatory.
- The above experimental arrangement is equivalent to an LC circuit, where the inductance is that of the loop and the capacitance is due to the spherical electrodes.
- Electromagnetic waves are radiated at very high frequency (≈ 100 MHz) as a result of oscillation of free charges in the loop.
- Hertz was able to detect these waves using a single loop of wire with its own spark gap (the receiver).
- Sparks were induced across the gap of the receiving electrodes when the frequency of the receiver was adjusted to match that of the transmitter.

PROPERTIES OF ELECTROMAGNETIC WAVES

- The electric and magnetic fields satisfy the following wave equations, which can be obtained from Maxwell's third and fourth equations.

$$\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{and} \quad \frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}$$

- Electromagnetic waves travel through vacuum with the speed of light c, where

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \text{ m/s}$$

- The electric and magnetic fields of an electromagnetic wave are prependicular to each other and also perpendicular to the direction of wave propagation. Hence, these are trnasverse waves.
- The instantaneous magnitudes of $\vec{E}$ and $\vec{B}$ in an electromagnetic wave are related by the expression

$$\frac{E}{B} = c$$

- Electromagnetic waves carry energy. The rate of flow of energy crossing a unit area is described by the Poynting vector $\vec{S}$. Where $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

- Electromagnetic waves carry momentum and hence can exert pressure(P) on surfaces,which is known as radiation pressure. For an electromagnetic wave with Poynting vector $\vec{S}$, incident upon a perfectly absorbing surface $P = \frac{S}{c}$
  and if incident upon a perfectly reflecting surface $P = \frac{2S}{c}$
- The electric and magnetic fields of a sinusoidal plane electromagnetic wave propagating in the positive x-direction can also be written as
  $$E = E_m \sin(kx - \omega t) \text{ and } \qquad B = B_m \sin(kx - \omega t)$$
  where ω is the angular frequency of the wave and k is wave number which are given by
  $$\omega = 2\pi f \quad \text{and} \quad k = \frac{2\pi}{\lambda}$$
- The intensity of a sinusoidal plane electro-magnetic wave is defined as the average value of Poynting vector taken over one cycle. $S_{av} = \frac{E_m B_m}{2\mu_0} = \frac{E_m^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_m^2$
- The fundamental sources of electromagnetic waves are accelerating electric charges. For examples radio waves emitted by an antenna aries from the continuous oscillations (and hence acceleration) of charges within the antenna structure.
- Electromagnetic waves obey the principle of superposition.
- The electric vector of an electromagnetic field is responsible for all optical effects. For this reason electric vector is also called a light vector.

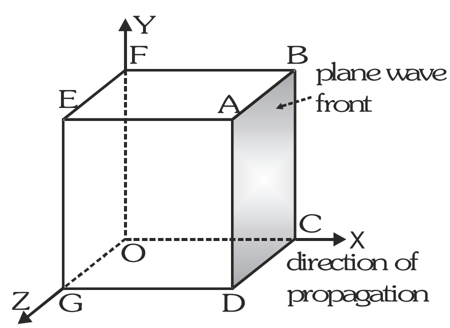
TRANSVERSE NATURE OF ELECTROMAGNETIC WAVES

Maxwell showed that a changing electric field produces a changing magnetic field and vice-versa. This alternate production of time 'varying electric and magnetic fields gives rise to the propagation of electromagnetic waves.

The variation of electric field ($\vec{E}$) and magnetic field ($\vec{B}$) are mutually perpendicular to each other as well as to the direction of the propagation of the wave i.e., the electromagnetic waves are transverse in nature.

Proof :

Consider a plane electromagnetic wave travelling along X-direction with its wave front in the Y–Z plane and ABCD is its portion at time t. The values of electric field and magnetic field to the left of ABCD will depend on x and t (and not on y and z as the wave under consideration is a plane wave propagating in x direction.



According to Gauss' law, the total electric flux across the parallelopiped' ABCDOEFG is zero because it does not enclose any charge. i.e. $\oint \vec{E}.\vec{dS} = 0$

or $$\oint_{ABCD} \vec{E}.\vec{dS} + \oint_{EFOG} \vec{E}.\vec{dS} + \oint_{ADGE} \vec{E}.\vec{dS} + \oint_{BCOF} \vec{E}.\vec{dS} + \oint_{OCDG} \vec{E}.\vec{dS} + \oint_{FBAE} \vec{E}.\vec{dS} = 0 \qquad ...(i)$$

since electric field $\vec{E}$ does not depend on y and z, so the contribution to the electric flux coming from the faces normal to y and z axes cancel out in pairs.

i.e. $$\oint_{OCDG} \vec{E}.\vec{dS} + \oint_{FBAE} \vec{E}.\vec{dS} = 0 \qquad ... (ii)$$

and $\oint_{ADGE}\vec{E}.\vec{dS}+\oint_{BCOF}\vec{E}.\vec{dS}=0$ ... (iii)

Using equation (ii) and (iii) in equation (i), we get

$$\oint_{ABCD}\vec{E}.\vec{dS}+\oint_{EFOG}\vec{E}.\vec{dS}=0 \quad \text{...(iv)}$$

Now $\oint_{ABCD}\vec{E}.\vec{dS}=\oint_{ABCD}E_x.dS\cos 0=\oint_{ABCD}E_x dS=E_x\int_{ABCD}dS$ ($\because$ $\vec{E}_x$ is parallel to $\vec{dS}$)

$= E_x$ area of face ABCD $= E_x S$ ... (v)

and $\oint_{EFOG}\vec{E'}.\vec{dS}=\oint_{EFOG}E'_x\,dS\cos 180°=E'_x\int_{EFOG}dS$ ($\because$ $\vec{E'}_x$ is antiparallel to $\vec{dS}$)

$= E'_x$ area of faceEFOG $= E'_x S$ ... (vi)

where, E_x and E'_x are the x-components of electric field on the faces ABCD and EFOG respectively.

Substituting the values of equations (v) and (vi) in equation (iv), we get

$E_xS - E_x'S = 0$ or $S(E_x - E_x') = 0$

$\because$ $S \neq 0$

$\therefore$ $E_x - E_x' = 0$ or $\boxed{E'_x = E_x}$

This equation shows that the value of the x-component of electric field does not change with time. In other words, electric field along x-axis is static.

Since the static electric field cannot propagate the wave, hence the electric field parallel to the direction of the propagation of the wave is zero.

i.e. $E_x' = E_x = 0$

It means, electric field is perpendicular to the direction of propagation of the wave.

similarly, it can be proved that the magnetic field is perpendicular to the direction of the propagationof the wave.

Since both electric and magnetic fields are perpendicualr to the direction of the propagation of the wave, so electromagnetic wave is transverse in nature.

GOLDEN KEY POINTS

- When a capacitor is connected across the battery through the connecting wires there is flow of conduction current, while thorugh the gap between the plates of capacitor,there is flow of displacement current.
- Maxwell's equation are mathematical formulation of Gauss's law in electrostatics (I) Gauss's law in electromagnetism (II) foradays law of electromagnetic induction (III) and Ampere's circuital law (IV)
- Frequency of electromagnetic waves is its inherent characterstic, when an electromagnetic wave travels from one medium to another, its wavelength changes but frequency remains unchanged.
- Ozone layer absorbs the ultra-violet rays from the sun and these prevents them from producing harmful effect on living organisms on the earth. Further it traps the infra-red rays and prevents them from escaping the surface of earth. It helps to keeps the earth's at atmosphere warm

Various parts of electromagnetic spectrum

S. No.	Radiation	Discover	How	Wavelength Range produced	Frequency range	Energy	Properties range	Application
1.	γ-Rays	Henry Becquerel and Madam Cuire	Due to decay of radioactive nuclei.	10^{-14}m to 10^{-10}m	3×10^{22}Hz to 3×10^{18}	10^{7}eV-10^{4} eV	(a) High penetrating power (b) Uncharged (c) Low ionising power	(a)Gives Information on nuclear structure (b) Medical treatment etc.
2.	X-Ray	Roentgen	Due to collisions of high energy electrons with heavy targets	6×10^{-12}m to 10^{-9}m	5×10^{19} Hz to 3×10^{17}Hz	2.4×10^{5}eV to 1.2×10^{3}eV	(a)Low Penetrating power (b) other properties similar to γ-rays except wavelength	(a) Medical diagnosis and treatment (b) Study of crystal structure (c) Industrial radiography
3.	Ultraviolet Rays	Ritter	By ionised gases, sun lamp spark etc.	6×10^{-10}m to 3.8×10^{-7}m	3×10^{17} Hz to 5×10^{19} Hz	2×10^{3}eV to 3eV	(a) All properties of light (b) Photoelectric effect	(a) To detect adulteration, writing and signature (b) Sterilization of water due to its destructive action on bacteria
4.	Visible light	Newton	Outer orbit electron transitions in atoms, gas discharge tube, incandescent solids and liquids.	3.8×10^{-7} m to 7.8×10^{-7}m	8×10^{14}Hz to 4×10^{14}Hz	3.2 eV to 1.6 eV	(a) Sensitive to human eye	(a)To see objects (b) To study molecular structure
	Subparts of visible spectrum							
	(a) Violet			3.9×10^{-7}m to 4.55×10^{-7}m	7.69×10^{14}Hz to 6.59×10^{14}Hz			
	(b) Blue			4.55×10^{-7}m to 4.92×10^{-7}m	6.59×10^{14}Hz to 6.10×10^{14}Hz			
	(c) Green			4.92×10^{-7}m to 5.77×10^{-7}m	6.10×10^{14}Hz to 5.20×10^{14}Hz			
	(d) Yellow			5.77×10^{-7}m to 5.97×10^{-7}m	5.20×10^{14}Hz to 5.03×10^{14}Hz			
	(e) Orange			5.97×10^{-7}m to 6.22×10^{-7}m	5.03×10^{14}Hz to 4.82×10^{14}Hz			
	(f) Red			6.22×10^{-7}m to 7.80×10^{-7}m	4.82×10^{14}Hz to 3.84×10^{14}Hz			

S. No.	Radiation	Discover	How producted	Wavelength Range	Frequency range range	Energy	Properties	Application
5.	Infra-Red waves	Willam Herschell	(a) Rearrangement of outer orbital electrons in atoms and molecules. (b) ChangeE of molecular vibrational and rotational energies (c) By bodies at high temperature.	7.8×10^{-7}m to 10^{-3}m	4×10^{14}Hz to 3×10^{11}Hz	1.6eV to 10^{-3}eV	(a) Thermal effect (b) All properties simillar to those of light except λ	(a) Used in industry, medicine and astronomy (b) Used for fog or haze photography (c) Elucidating molecular structure.
6.	Microwaves	Hertz	Special electronic devices such as klystron tube	10^{-3} to 0.3m	3×10^{11} Hz to 10^{9} Hz	10^{-3}ev to 10^{-5}eV	(a) Phenomena of reflection, refraction and diffraction	(a) Radar and telecommunication. (b) Analysis of fine details of molecular structure
7.	Radio waves Subparts of Radio-spectrum	Marconi	Oscillating circuits	0.3 to few kms.	10^{9}Hz to few Hz	10^{-3}eV to ≈ 0	(a) Exhibit waves like properties more than particle like properties.	(a) Radio communication
(A)	Super High Frequency (a) SHF Ultra High Frequency (b) UHF Very High Frequency (c) VHF			0.01m to 0.1m 0.1 m to 1m 1 m to 10 m	3×10^{10}Hz to 3×10^{9}Hz 3×10^{9}Hz to 3×10^{8}Hz 3×10^{8}Hz to 3×10^{7}Hz		Radar, Radio and satelite communication (Microwaves), Radar and Television broadcast short distance communication, Television communication.	
(B)	High Frequency (HF) Medium Frequency (MF) Low Frequency (LF) Very Low Frequency (VLF)			10m to 100 m 100 m to 1000 m 1000 m to 10000 m 10000 m to 30000 m	3×10^{7}Hz to 3×10^{6}Hz 3×10^{6}Hz to 3×10^{5}Hz 3×10^{5}Hz to 3×10^{4}Hz 3×10^{4}Hz to 10^{4}Hz		Medium distance communication Telephone communication, Marine and navigation use, long range communication. Long distance communication.	

SOME WORKED OUT EXAMPLES

Example#1

In a plane electomagnetic wave, the electric field oscillates sinusoidally at a frequency of $2 \quad 10^{10}$ Hz and amplitude 48 V/m. The amplitude of oscillating magnetic field will be –

Solution

Oscillating magnetic field $B = \frac{E}{c} = \frac{48}{3 \times 10^{8}} = 16 \times 10^{-8}$ Wb/m^2

Example#2

In the above problem, the wavelength of the wave will be –

Solution

Wavelength of electromagnetic wave $\lambda = \frac{c}{\nu} = \frac{3 \times 10^{8}}{2 \times 10^{10}} = 1.5 \times 10^{-2} = 1.5\,\text{cm}$

Example#3

A point source of electromagnetic radiation has an average power output of 800W. The maximum value of electric field at a distance 3.5 m from the source will be –

Solution

Intensity of electromagnetic wave given is by $I = \frac{P_{av}}{4\pi r^2} = \frac{E_m^2}{2\mu_o c}$

$$E_m = \sqrt{\frac{\mu_o c P_{av}}{2\pi r^2}} = \sqrt{\frac{(4\pi \times 10^{-7}) \times (3 \times 10^{8}) \times 800}{2\pi \times (3.5)^2}} = 62.6 \text{ V/m}$$

Example#4

In the above problem, the maximum value of magnetic field will be –

Solution

The maximum value of the magnetic field is given by $B_m = \frac{E_m}{c} = \frac{62.6}{3 \times 10^{8}} = 2.09 \quad 10^{-7}$ T

Example#5

What should be the height of transmitting antenna if the T.V. telecast is to cover a radius of 128 km ?

Solution

Height of transmitting antenna $h = \frac{d^2}{2R_e} = \frac{(128 \times 10^3)^2}{2 \times 6\cdot4 \times 10^6} = 1280\,\text{m}$

Example#6

The area to be covered for T.V. telecast is doubled, then the height of transmitting antenna (T.V. tower) will have to be –

Solution

The area of transmission of surrounding the T.V. tower $A = \pi d^2 = \pi(2hRe) \quad A \propto h$

Example#7

In an electomagnetic wave, the amplitude of electric field is I V/m. The frequency of wave is 5 10^{14} Hz. The wave is propagating along z-axis. The average energy density of electric field, in Joule/m^3, will be

Solution

Average energy dendity is given by

$$u_E = \frac{1}{2}\varepsilon_o E^2 = \frac{1}{2}\varepsilon_o\left(\frac{E_o}{\sqrt{2}}\right)^2 = \frac{1}{4}\varepsilon_o E_o^2 = \frac{1}{4}\times 8.85\times 10^{-12}\times(1)^2 = 2.2 \quad 10^{-12}\ \text{J/m}^2$$

Example#8

A T.V. tower has a height of 100 m. How much population is covered by T.V. broadcast, if the average population density around the tower is 1000/km^2 ?

Solution

Radius of the area coved by T.V. telecast $d = \sqrt{2hR_e}$

Total population covered $= \pi d^2$ population density $= 2\pi hR_e$ polulation density

$$= 2 \quad 3.14 \quad 100 \quad 6.4 \quad 10^6 \quad \frac{1000}{10^6} = 39.503 \quad 10^5$$

Example#9

An electomagnetic radiation has an energy 14.4 KeV. To which region of electromagnetic spectum does it belong ?

Solution

$$\lambda = \frac{hc}{E} = \frac{6.6\times 10^{-34}\times 3\times 10^8}{14.4\times 10^3\times 1.6\times 10^{-19}} = 0.8 \quad 10^{-10}\ \text{m} = 0.8\text{Å}$$

This wavelength belongs to X–ray region

EXERCISE 1 CHECK YOUR GRASP

1. If $\vec{E}$ and $\vec{B}$ are the electric and magnetic field vectors of electromagnetic waves then the direction of propagation of electromagnetic wave is along the direction of –

(1) $\vec{E}$ (2) $\vec{B}$ (3) $\vec{E}$ $\vec{B}$ (4) none of these

2. The electromagnetic waves do not transport -

(1) energy (2) charge (3) momentum (4) information

3. In an electromagnetic wave the average energy density is associated with –

(1) electric field only (2) magnetic field only
(3) equally with electric and magnetic fields (4) average energy density is zero

4. In an electromagnetic wave the average energy density associated with magnetic field will be

(1) $\frac{1}{2}LI^2$ (2) $\frac{B^2}{2\mu_0}$ (3) $\frac{1}{2}\mu_0 B^2$ (4) $\frac{1}{2}\frac{\mu_0}{B^2}$

5. In the above problem, the energy density associated with the electric field will be –

(1) $\frac{1}{2}CV^2$ (2) $\frac{1}{2}\frac{q^2}{C}$ (3) $\frac{1}{2}\frac{\varepsilon^2}{E}$ (4) $\frac{1}{2}\varepsilon_0 E^2$

6. In which part of earth's atmosphere is the ozone layer present ?

(1) troposphere (2) stratosphere (3) ionosphere (4) meosphere

7. The ozone layer is earth's atmosphere is crucial for human survival because it –

(1) has ions (2) reflects radio signals
(3) reflects ultraviolet rays (4) reflects infra red rays

8. The frequency from 3 10^9 Hz to 3 10^{10} Hz is –

(1) high frequency band (2) super high frequency band
(3) ultra high frequency band (4) very high frequency band

9. The frequency from 3 to 30 MHz is known as –

(1) audio band (2) medium frequency band
(3) very high frequency band (4) high frequency band

10. The AM range of radiowaves have frequency –

(1) less than 30 MHz (2) more than 30 MHz (3) less than 20000Hz (4) more than 20000Hz

11. Select wrong statement from the following for EMW -

(1) are transverse (2) travel with same speed in all medium
(3) travel with the speed of light (4) are produced by accelerating charge

12. The nature of electromagnetic wave is –
(1) longitudinal (2) longitudinal stationary (3) transverse (4) transverse stationary

13. Which of the following are not electromagnetic waves ? **[AIEEE–2002]**
(1) Cosmic-rays (2) γ-rays (3) β-rays (4) X-rays

14. The rms value of the electric field of the light coming from the sun is 720 N/C. The average total energy density of the electromagnetic wave is- **[AIEEE–2006]**
(1) 4.58 10^{-6} J/m^3 (2) 6.37 10^{-9} J/m^3 (3) 81.35 10^{-12} J/m^3 (4) 3.3 10^{-3} J/m^3

15. An electromagnetic wave in vacuum has the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarizaiton is given by $\vec{X}$ and that of wave propagation by $\vec{k}$. Then :- **[AIEEE - 2012]**
(1) $\vec{X} || \vec{E}$ and $\vec{k} || \vec{B} \times \vec{E}$ (2) $\vec{X} || \vec{B}$ and $\vec{k} || \vec{B} \times \vec{E}$
(3) $\vec{X} || \vec{E}$ and $\vec{k} || \vec{E} \times \vec{B}$ (4) $\vec{X} || \vec{B}$ and $\vec{k} || \vec{E} \times \vec{B}$

16. An electromagnetic wave with frequency ω and wavelength λ travels in the +y direction. Its magnetic field is along _ x axis. The vector equation for the associated electric field (of amplitude E_0) is :- **[AIEEE–2012 (Online)]**
(1) $\vec{E} = E_0 \cos\left(\omega t - \frac{2\pi}{\lambda} y\right) \hat{x}$ (2) $\vec{E} = -E_0 \cos\left(\omega t + \frac{2\pi}{\lambda} y\right) \hat{x}$
(3) $\vec{E} = -E_0 \cos\left(\omega t + \frac{2\pi}{\lambda} y\right) \hat{z}$ (4) $\vec{E} = E_0 \cos\left(\omega t - \frac{2\pi}{\lambda} y\right) \hat{z}$

ANSWER KEY EXERCISE

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	3	2	3	2	4	2	3	2	2	1	2	3	1, 3	1	3	4